Finite density [might well be easier] at finite temperature

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Experiments with imaginary chemical potential and Glasgow method carried out in two interrelated models – four dimensional QCD in the infinite coupling limit, and one dimensional QCD – support the point of view expressed by the title.

The profound differences of dense, cold matter, and hot, moderately dense, hadron gas might be reflected by different performances of numerical approaches to finite density QCD. In particular, the enhancement of the fluctuations near T_c should give better chances either to the Glasgow method [1], by ameliorating the overlap problem, and to the imaginary chemical potential[2], which does not systematically bias the ensemble and thus needs large response to infinitesimal μ changes near $\mu = 0$.

The fluctuations of baryons are measured by the baryon number susceptibility

$$\chi(T,\mu) = \partial\rho(\mu,T)/\partial\mu = \partial^2 logZ(\mu,T)/\partial\mu^2 \quad (1)$$

Lattice results [5] indicate the range where $\chi(T,\mu=0)$ is significantly different from zero : this is the candidate region for performing finite density calculations below T_c – a narrow, but not minuscule interval.

As $Z(\mu)$ (we shall explicit only the μ dependence) is an even function of μ, we have:

$$logZ(\mu) = K + a\mu^2 + b(\mu^4) + O(\mu^6) \quad (2)$$
$$logZ(\nu) = K - a\nu^2 + b(\nu^4) + O(\nu^6) \quad (3)$$

where $\nu = i\mu$. Note that $a = \chi(T,0)$. Consider

$$a_{eff}(\mu) = (logZ(\mu) - logZ(\nu))/2\mu^2 \quad (4)$$
$$b_{eff}(\mu) = (logZ(\mu) + logZ(\nu) - 2K)/2\mu^4 \quad (5)$$

A fourth order polynomial describes the data where a_{eff} and b_{eff} do not depend on μ.

The strong coupling limit of QCD displays confinement and chiral symmetry breaking, and, besides having produced interesting, semiqualitative results on the phase diagram, offers a nice test bed for numerical methods.

Consider the effective potential V_{eff} (related to Z as $Z = (\int V_{eff} d\lambda)^{V_s}$) in the $g = \infty$ limit of QCD [3]:

$$V_{eff}(\lambda,\mu) = 2cosh(rN_tN_c\mu) + sinh[(N_t+1)N_c\lambda]/sinh(N_t\lambda) \quad (6)$$

where the variational parameter λ (essentially, $< \bar{\psi}\psi >$) is to be determined by a minimum condition, N_t is the number of points in time direction, N_c the number of colors (which we fix: $N_c = 3$), r the asymmetry factor a_τ/a_β, and rN_t is the inverse temperature. Note that formally $V_{eff}(\lambda,\mu)$ is an one dimensional partition function which "remembers" its four dimensional origin via λ.

For a purely imaginary chemical potential, $cosh(rN_tN_c\mu) \to cos(rN_tN_c\mu)$. We see the expected periodicity $2\pi/(rN_tN_c)$, and we note that the chemical potential term can be ignored for large N_t (zero temperature). Indeed, we have verified that in this limit any dependence on the chemical potential is lost [4], Fig.1 : at $T = 0$ $< \bar{\psi}\psi >$ as a function of (complex) μ is a constant in the half plane $\Re(\mu) < \mu_c$, μ_c being the (real) critical chemical potential. By increasing the temperature, the effective potential changes with imaginary μ, and the chiral condensate *increases* with imaginary chemical potential, as it should. This behaviour is shown in Fig. 2, where we plot the chiral condensate as a function of a real, and a purely imaginary μ rather close to T_c. We note that the fourth order polynomial behaviour anticipated above holds in a rather large

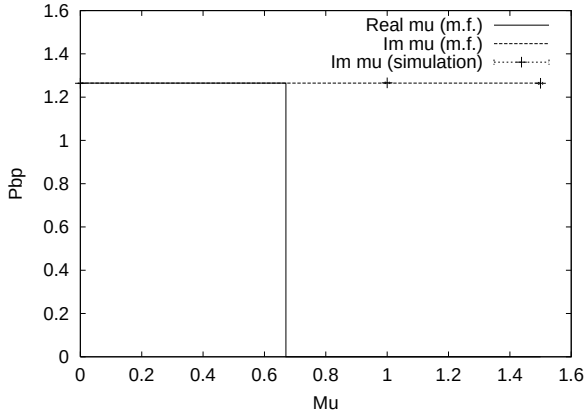

Figure 1. $< \bar{\psi}\psi >$ as a function of real and imaginary chemical potential (a constant) in the zero temperature limit. The only relevant quantity – μ_c– is not amenable to an analytic continuation from imaginary μ.

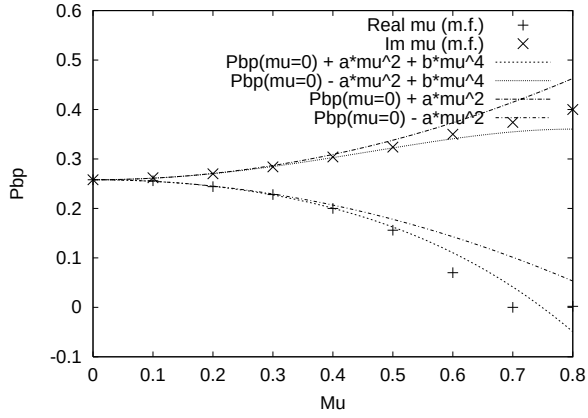

Figure 2. $< \bar{\psi}\psi >$ as a function of real (crosses) and imaginary chemical potential (plus'es) for T slightly below T_c. A polynomial representation (eqs. (2) and (3)) describes well either data set. a and b are computed as in eqs. (4) and (5).

range of chemical potential, and that a simple second order polinomial works fine as well.

In the same hot region reweighting too might prove useful. To check that, we have tested the Glasgow method in one dimensional QCD[6], a solvable model without SSB, but with baryons, whose partition function is formally identical to the effective potential above. The crucial difference comes from the meaning of the parameter λ which here is simply $sh^{-1}m$. The bare mass m plays the rôle of an explicit breaking term, and $< \bar{\psi}\psi > \neq 0$ when $m \neq 0$. Fixing $N_c = 3$, the relevant parameter is $N_t\lambda$, which is the analogous of an inverse temperature.

At large "temperature" (small $N_t\lambda$) we have verified that the Glasgow method reproduces the exact results [7], thus supporting the idea that reweighting methods can be successfully used in that regime. We would like to stress that this is a non trivial results since at large temperature quenched and exact results are distinct [6]. This contrasts with the low temperature regime of the same model, where the quenched approximation is apparently exact, and the success of the Glasgow method is then guaranteed a priori (as the Glasgow method reproduces the quenched results for small statistics).

Consider the zeros of the determinant, which control the singularities of the quenched model, and those of the partition function, the Lee Yang zeros controlling the singularities of the full model, in the complex plane e^{μ}. These $N_t \times N_c$ zeros can be easily calculated using the results of [6] and are shown in Figs. 3. The Glasgow method (Fig. 4) reproduces them correctly, as expected given that Z itself approaches the correct value as mentioned above. The smoothening of the transition in the full model noted in [6] can be clearly read off Figs. 3, 4 : the zeros of Z move away from the real axis. It should be noticed, however, that in this case there is no pathological onset (no Goldstone particle here!): the Glasgow method only needs rearranging zeros on circular patterns. It is reasonable to hope that the same happens in high temperature, four dimensional QCD. From Fig. 3 we also understand why the quenched approximation is apparently working on long lattices [6]: the zeros will become dense

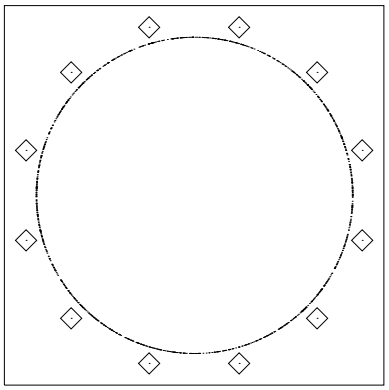

Figure 3. Distribution of the zeros of the determinant (100 configurations) and the exact results for the zeros of the partition function Z with $N_t = 4$ (diamonds) for one dimensional QCD.

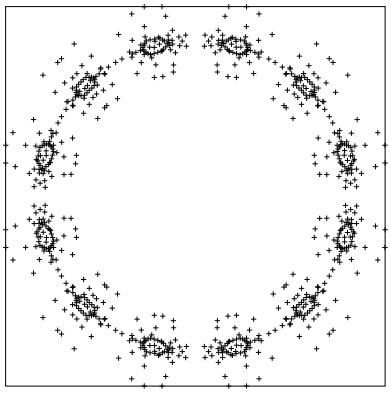

Figure 4. The zeros of $Z =< Det >$ from the Glasgow method approach the exact pattern Fig. 3. Correspondingly, number density and chiral condensate become nearly exact.

on the same circle where the zeros of the determinant are randomly distributed. An important difference between the full and quenched transition however remains: only the zeros of the partition functions are fixed by symmetries. Hence, only the nearest zero of Z (and not that of the determinant) has a calculable distance from the real axes $\simeq \mu_c/(3N_t)$, fulfilling a Lee-Yang–like scaling towards the $N_t = \infty$ singularity.

We have given examples of clear signals obtained by use of an imaginary chemical potential, and successful calculations with the Glasgow method in toy models related with QCD. We have suggested a possible simple line of analysis which combines and cross checks results from both approaches.

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